# How to promote learning in problem-solving?

Laine, Anu

WTM-Verlag
2019

Laine , A 2019 , How to promote learning in problem-solving? in A Kuzle , I Gebel & B Rott (eds) , Implementation Research on Problem Solving in School Settings : Proceedings of the 2018 Joint conference of ProMath and the GDM Working Group on Problem Solving . Ars inveniendi et dejudicandi , vol. 13 , WTM-Verlag , Münster , pp. 3-18 , Joint Conference of Problem Solving in Mathematics Education group (ProMath) and the GDM Working Group on Problem Solving 2018 , Potsdam , Germany , 29/08/2018 .

http://hdl.handle.net/10138/310463

unspecified
acceptedVersion

# HOW TO PROMOTE LEARNING IN PROBLEM-SOLVING

Anu Laine

University of Helsinki, Finland

*This article is based on my plenary talk at the jjoint conference of ProMath and the GDM working group on problem-solving in 2018 The aim of this article is to consider teaching and learning problem-solving from different perspectives. I will consider the connection between 1) teacher's actions and pupils' solutions and 2) teacher's actions and pupils' affective reactions. Safe and supportive emotional atmosphere is base for students' learning, improved performance and attitudes towards mathematics and learning*. *Interaction among pupils and between pupils and the teacher is of crucial importance in the development of the positive emotional atmosphere. Teacher has a central role both in constructing emotional atmosphere and in offering cognitive support that pupils need in order to reach higher-level solutions. Teachers need to use activating guidance, i.e., ask good questions based on pupils' solutions. Balancing between too much and too little guidance is not easy.*

## INTRODUCTION

This article is based on my plenary talk at the joint conference of ProMath and the GDM working group on problem-solving in 2018. My talk was inspired on many observations. I have gained during two different projects: (1) Finland – Chile research project (funded by the Academy of Finland and Chilean CONICYT, principal investigator professor Erkki Pehkonen, 2010-2013), and (2) Luma Finland project "*Understanding for problem-solving*" (funded by the Ministry of Education and Culture, 2014-2019, principal investigator. Anu Laine). The aim of (1) was to develop a model for improving the level of pupils' mathematical understanding by using open problem tasks in mathematics teaching. It was a follow up study with an experimental group of 10 teachers and their pupils from 3rd to 5th grade. The aim of (2) is to increase primary school teachers' abilities to use problem-solving in their teaching.

### Why problem-solving and open problems?

Why is it then important to teach problem-solving? Problem-solving is a central tool to develop mathematical thinking (e.g., Schoenfeld, 1985, 1992). Using problem-solving in teaching also improves pupils' mathematical knowledge (Stigler & Hiebert, 2004). In addition, the Finnish National Curriculum (FNBE, 2004, 2014) obligates to teach problem-solving at all levels of education. The aim

**In I. Gebel, A. Kuzle, & B. Rott (Eds.), Proceedings of the 2018 Joint Conference of ProMath and the GDM Working Group on Problem-solving (pp. x–y). Münster: WTM-Verlag.**

in primary education is to 1) develop logical, accurate and creative mathematical thinking, 2) learn to explain thinking by talking, writing and drawing, and 3) learn different problem-solving strategies (FNBE, 2004, 2014).

Most tasks and problems are either open or closed. If the task is closed, then the answer is determinate, the goal is specific, there is only one possible method to solve it, and the task cannot be extended (Yeo, 2017). The openness in problems can appear in different ways (Pehkonen, 1997). Open problem-solving tasks can have an open goal (e.g., Find as many as you can.) or open starting point (e.g., What kind of cake should I bake so that it is enough for 10 persons?). Open problems can also be methodically open, and facilitate different solution methods and strategies.

Why then open problems? Open problems resemble more real life problems like baking a cake. They most likely activate pupils on different levels, help to accept more than one solution, and help to rehearse verbalizing own thinking.

## Teacher's role during a problem-solving lesson

A problem-solving lesson can be divided into three phases: launch phase, explore phase, and discussion and summarize phase (e.g., Stein, Engle, Smith, & Hughes, 2008). The teacher has a central role in all phases when providing both cognitive and emotional support. All students need support in order to reach their best.

*In launch phase*, the teacher introduces the problem and the concepts related to the task so that the students understand what they are supposed to do (Stein et al., 2008). During launch phase, the teacher also motivates pupils to solve and ponder on the problem. In addition, it is central that the teacher creates a safe atmosphere where students dare to try different things out and make mistakes along the way (Pekrun & Stephens, 2010.) This is important in all phases.

*During explore phase* students work on problems either individually or in small groups. The teacher gives them cognitive support by guiding and deepening students' thinking. The teacher asks students good questions (e.g., Sahin & Kulm, 2008) in order to find out their thinking and to support the problem-solving process. The teacher's task is to guide students toward right direction without revealing the solution. The students need also teacher's encouragement when they feel frustrated (Hannula, 2015).

We have identified two dimensions for the quality of teacher's guidance (Laine, Ahtee, Näveri, Pehkonen, & Hannula, 2017). One dimension is about focusing on the relevant ideas. The guidance can be either deep guidance or surface-level guidance depending on how much teacher's guidance is based on understanding student's idea. The other aspect is about supporting student's own thinking. The guidance can be either activating guidance or inactivating guidance depending on how much the teacher is able to activate student's own thinking. *The discuss and summarize phase* is a central part of mathematics lesson as it is rich with learning opportunities (Pólya, 1945). At this point of the lesson, the students present and discuss their solutions and their solution methods. The teacher's role is to pay pupils' attention to central concepts and strengthen their mathematical thinking. It is also important, that the teacher maintains positive atmosphere which encourages all pupils to participate in the discussion (Stein et al., 2008.)

### Aim of the paper

The aim of this paper is to consider teaching and learning problem-solving from different perspectives. Firstly, I consider the connection between the teacher's actions and pupils' solutions. Secondly, I explore the connection between the teacher's actions and pupils' affective reactions

## THE CONNECTION BETWEEN TEACHER'S ACTIONS AND PUPILS' SOLUTIONS

In this section, I present four studies, that I have conducted with my colleagues to articulate the connection between teacher's actions and pupils' solutions.

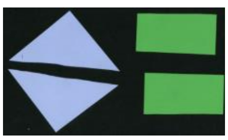
### Teachers' actions supporting higher-level solutions

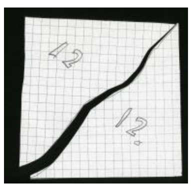
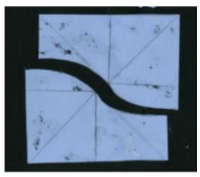
The first one published in International Journal of Science and Mathematics Education (Laine et al., 2017) tries to find out what factors in the teachers' actions seem to have an effect for pupils to reach higher-level solutions.

Fourth grade pupils (N = 86) and their seven teachers, Ann (12 pupils), Beatrice (14 pupils), Cecilia (16 pupils), Danielle (13 pupils), Eve (16 pupils), Fatima (7 pupils) and Gabrielle (8 pupils), received an open problem: "*Divide a square with a line into two exactly equal pieces*." This means that the pieces should be convergent. The pupils spent one 45 minutes lesson on this problem.

Pupils' solutions to this problem were divided in five solution levels: no solution (level 0), basic level (level 1), straight line (level 2), curved line (level 3), and middle-point solution (level 4). Finding out basic solution was easy for the students, but the other solutions demanded creativity and giving up from line-symmetrical thinking. In level 4 –solutions, the middle-point of the square was seen as essential part of the solutions, so that the dividing lines – straight or curved – are symmetrical in relation to the middle-point. The pupils' solution levels and examples of different solutions are presented in Table 1.

| No solution | Basic level | Straight line | Curved line | Middle-point |
|---|---|---|---|---|
| Level 0 | Level 1 | Level 2 | Level 3 | Level 4 |
| 1 (1%) | 33 (38%) | 21 (24%) | 18 (21%) | 13 (15%) |



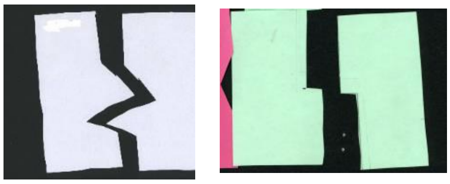


*Table 1*. Pupils' solution levels in "Divide a square" problem.

Pupils' solution levels between classes differed greatly. Moreover, there were also differences in the ways the teachers introduced the task in the launch phase. Some of the teachers used a model i.e. gave an example to help the pupils to understand what dividing into two pieces means. For example, Cecilia divided a triangle into two pieces. The way teachers guided pupils' work during explore phase differed likewise. Some of the teachers (Ann and Beatrice) used activating support, i.e., asked questions about pupil's solution that helped the pupil to proceed. Some of them (Cecilia, Danielle, Eve and Gabrielle) used commenting support, i.e., they encouraged the pupils to proceed by saying "Good work." Fatima) did not provide any guidance whatsoever. There were also differences with respect to the materials teachers used during the lesson. Some of them used for example scissors and grid paper. Moreover, some of the teachers (Beatrice, Cecilia, Danielle, Eve and Gabrielle) also revealed some solutions during the lesson. The pupils had performed a mathematics test at the beginning of the project. Gabrielle's class was weaker than the other classes. There were not significantdifferences in mathematical knowledge between other classes. A summary of factors that might have had an impact on pupils' solutions are presented in Table 2.

| | **Level of solution** | **Ann** | **Beatrice** | **Cecilia** | **Danielle** | **Eve** | **Fatima** | **Gabrielle** |
|---|---|---|---|---|---|---|---|---|
| **Pupils' performances** | **0** | 0% | 0% | 0% | 8% | 0% | 0% | 0% |
| | **1** | 42% | 14% | 37% | 0% | 50% | 86% | 75% |
| | **2** | 0% | 29% | 13% | 58% | 37% | 0% | 0% |
| | **3** | 33% | 14% | 37% | 34% | 13% | 14% | 0% |
| | **4** | 25% | 43% | 13% | 0% | 0% | 0% | 25% |
| **Pre-test (class average)** | | 7.1 | 8.4 | 7.8 | 7.3 | 6.2 | 6.6 | 2.9 |
| **Launch phase** | | M | M | M | M | NoM | InM | InM |
| **Explore phase** | | AS | AS | CS | CS | CS | NS | CS |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Materials** | S, GP | S, GP,BP | S, BP | S, BP | S, BP | S, BP | S, BP |
| **Teacher revealed solutions** | NR | SM | L | L | L | NR | SM |

*Table 2.* Distribution on pupils' solutions and other information according to teachers (M = model, NoM = no model, IM = incorrect model; teacher's action during the Explore phase: AS = activating support, CS = commenting support, NS = no support; ; materials used during the lesson: S = scissors, BP = blank paper, GP = grid paper; teacher's way of revealing the solution: SM = solution model, L= line is not straight, NR = no revelation).

The way in which the teachers introduced the task seemed to have a central consequence: Most of the pupils in Eve's (no model), Gabrielle's (incorrect model) and Fatima's (incorrect model) classes remained at level 1. It is also interesting to notice that all pupils except one reached at least level 2 in Danielle's class. It is possible that her circle model with many diagonals at the beginning of the lesson helped the pupils to invent level 2 solutions. In addition, the choice of materials influenced the results. For instance, Ann and Beatrice provided grid paper, and in these classes, more students invented curved-lined middle-point solutions.

Also, the way in which the teachers guided their pupils was relevant Fatima did not provide any guidance and six of her eight pupils (86%) remained at level 1. Guidance with activating support (i.e. asking questions about the pupils' solution that help the pupil to proceed) was effective: The pupils in Ann's and Beatrice's classes invented more level 4 solutions than the pupils in other teachers' classes. Unfortunately, Ann's class was restless and she was not able to guide all the pupils. In addition, commenting support (i.e. encouraging pupils to proceed on a general level) seemed to help in Cecilia's and Danielle's classrooms. Thus, pupils may try harder when the teacher encourages them, and shows interest in their work.

## The connection between mathematical knowledge and problem solving performance

We wanted to examine closer pupils' performance in basic calculation test and its relation to the solution levels in "Divide a square" problem. This part of the plenary was based on the article published in NMI-bulletin (Laine, Näveri, Ahtee, & Pehkonen, 2016). We divided the pupils' performance in basic calculation in three categories (weak, average and good) and made a cross tabulation in relation to their performance in the problem-solving task (different solution levels) as shown in Table 3.

| Pre-test: basic calculations | Pupils' performance in problem-solving task | | | | Total |
|---|---|---|---|---|---|
| | Level 1 | Level 2 | Level 3 | Level 4 | |
| Weak performance: 1–2 points | 12 (67 %) | 2 (11 %) | 2 (11 %) | 2 (11 %) | 18 (100 %) |
| Average performance: 3 points | 10 (45 %) | 2 (9 %) | 5 (23 %) | 5 (23 % $^{T}$) | 22 (100 %) |
| Good performance: 4–5 points | 16 (41 %) | 15 (38 % $^{T}$) | 6 (15 %) | 2 (5 %) | 39 (100 %) |

*Table 3.* The relation between pupils' performance in basic calculation and solution levels in "Divide a square" problem. T = proportion bigger than expected.

There is a statistically significant connection between pupils' performance in basic calculations and the problem-solving task. Hence, the students with weaker calculation skills invented lower level solutions in the problem-solving task (see more Laine et al., 2016).We were also interested in the relationship between teacher's actions and pupils' performance, and, hence divided the teachers in two categories: teachers giving activating support (Ann and Beatrice) and teachers giving non-activating support (rest of the teachers). We looked for the relation between the classes with activating support and non-activating support with the solution levels in the problem-solving task (see Table 4).

| | | Level 1 | Level 2 | Levels 3–4 | Number of pupils |
|---|---|---|---|---|---|
| 1–2 points | Activating support | 50 % | 0 % | 50 % | 4 |
| | Non-activating support | 53 % | 10 % | 37 % | 14 |
| 3 points | Activating support | 40 % | 0 % | 60 % | 10 |
| | Non-activating support | 50 % | 17 % | 33 % | 12 |
| 4–5 points | Activating support | 18 % | 36 % | 45 % | 11 |
| | Non-activating support | 50 % | 39 % | 11 % | 28 |

*Table 4.* The relation between performance in basic calculations and problem-solving performance in classes with activating support (Ann and Beatrice) and non-activating support (rest).

Pupils who were provided activating support seemed to reach more level 3 and 4 solutions. Correspondingly, it seems that pupils in classes with non-activating support reached mostly levels 1 and 2 (see Table 5).

| | Level 1 | Level 2 | Levels 3–4 | Total |
|---|---|---|---|---|
| Non-activating support | 30 (56 %) | 15 (28 %) | 9 (16 % [A]) | 54 (100 %) |
| Activating support | 8 (32 %) | 4 (16 %) | 13 (52 % [T]) | 25 (100 %) |

*Table 5.* The relation between support and problem-solving performance. NB. A = proportion smaller than expected, custom standard remainder ≤ -2, T = proportion bigger than expected, custom standard remainder ≥ 2.

There is a statistically significant connection between support and pupils' problem-solving levels. This means that pupils who received activating support reached higher- level solutions in the problem-solving task (see more Laine et al., 2016). This implies that teacher's support is relevant for all pupils and that the support really helps pupils.

## Teachers' actions supporting written explanations

The connection between teachers' actions and pupils' performance was also studied in the third article published in LUMAT – International Journal on Math, Science and Technology Education (Laine, Ahtee, Näveri, Hannula, & Pehkonen, 2018). The aim of the study was to find out in what way are the teachers' actions with respect to fostering written explanations related to the explanations given by the pupils.

The pupils were solving arithmagons. An arithmagon is a triangle where the sum of the corner numbers is given in the middle of the sides (see Mason, Burton, & Stacey, 1982). The pupils were instructed as follows: "*Solve simplified arithmagons and invent a method how you can always solve the corner numbers when the numbers in the middle of the sides are given and two of these are equal*" (Figure 1.)

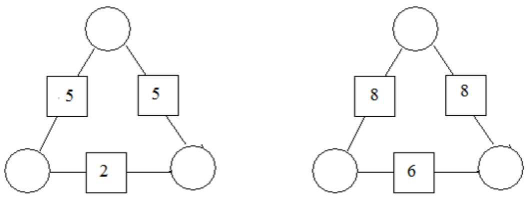



*Figure 1.* Examples of simplified arithmagons.

All pupils (N = 94) solved the problem. About half of the pupils (N = 41) gave some kind of written explanation. The answers were divided in four categories, as shown in Table 6. In the most profound explanations, category X.1 '*Two same numbers'*, the pupils had paid attention to the fact that these arithmagons contained two same numbers. In category X.2 '*Addition*' the pupils noted that they had used addition in their calculations when they tried to find corner numbers. The third category X.3 '*A vague expression*' contains the answers in which the pupils wrote that they just calculated. These expressions are more like descriptions than strategies used to find a solution to the problem. Most of the pupils did not provide any mathematical explanation, but just wrote that they did not know how to solve the problem, and these answers were included in category Y, '*No explanation*'.

| *Category* | | *Examples* | *Number of pupils* |
|---|---|---|---|
| X | Explanation | | 41 |
| X.1 | Two same numbers | *There are always two same numbers in the triangles.*<br>*I started by adding the topmost number, because this one number has to fit with two numbers.* | 13 |
| X.2 | Addition | *I just did + calculations.*<br>*I added the corner numbers because so I got the numbers in the sides.* | 16 |
| X.3 | A vague expression | *I just calculated.*<br>*Finally, I just understood it.* | 12 |
| Y | No explanation | *I don't know.* | 53 |

*Table 6.* The distribution of the pupils' explanations in the four categories with examples.

The teachers guided their pupils in different ways, and emphasized different matters (e.g., requested explanation or not, gave support for writing the explanation or not) in their instructions. In Table 7, a summary of how the teachers requested written explanations, how they supported the pupils to explain their solution, and the pupils' performances is given.

| | Katie | Sophie | Lily | Ruby | Julia | Eva | Sarah |
|---|---|---|---|---|---|---|---|
| Explanation requested | Yes | Yes | Yes | Different task | Different task | No | No |

| Support for explanation | Deep questioning | Questioning | Questioning | Questionin g | Deep questioning | No | No |
|---|---|---|---|---|---|---|---|
| X.1 Two same numbers | 11 | 0 | 0 | 2 | 0 | 0 | 0 |
| X.2 Addition | 4 | 2 | 8 | 0 | 2 | 0 | 0 |
| X.3 Vague expression | 1 | 8 | 3 | 0 | 0 | 0 | 0 |
| Y. No reasoning | 1 | 2 | 4 | 14 | 11 | 8 | 13 |
| Number of pupils | 17 | 12 | 15 | 16 | 13 | 8 | 13 |

*Table 7.* The cross-tabulation of the teachers' demand and support for written explanations, and the pupils' performance.

The study results imply that it is important that the teacher pays special attention to requiring explanation as a part of pupils' learning. In all classes where explanation was requested, the pupils wrote down their explanations. It is interesting that also in classes where the teachers (Ruby and Julia) requested explanation for different tasks (main arithmagon instead of simplified arithmagon with two same numbers), the pupils were able to write down the correct explanation. Additionally, teachers' questions helped pupils to write down their explanations. Especially deep questioning, which activated pupils' thinking, helped pupils to provide more details in their explanations.

## Different ways to teach problem-solving

The fourth article considers the connection of teacher's actions and pupils' solutions from teacher'sperspective. The article is under review in International Journal for Mathematics Teaching and Learning (Portaankorva-Koivisto, Laine, & Ahtee, under review). The aim of the study is to examine the different ways in which two primary teachers' beliefs, knowledge and practices interact when learning to teach mathematical problem-solving.

Both teachers, Alice and Bea, participated in Finland-Chile research project with their classes from 3$^{rd}$ to 5$^{th}$ grade. Once a month, they conducted a 45-minute mathematics lesson in which they used an open problem-solving task. They dealt with altogether 20 problems in their classes during three years. In addition, once a month the teachers had a meeting with the researchers. In these meetings, the researchers gave lectures about problem-solving and its teaching, discussed the implementation and experiences of the previous problem with the teachers and provided them with a new problem. Teachers were not given the solution to the task or any normative practices. Rather, the idea was that the teachers develop different practices with their classes (learning by doing) and learn from each other when discussing about the lessons together.

We interviewed Alice and Bea two times and examined video recordings from their lessons. By doing that, we were able to analyse their beliefs, knowledge and practices.

Alice did not place much value on mathematical rules and systematic thinking but appreciated hands-on materials. During the lessons she overlooked the mathematical content and let her pupils work freely. Pupils' guidance demands good content knowledge (e.g. Hill, Ball & Schilling, 2008), which Alice did not have. In the interview, she emphasized both emotional and cognitive support, but failed to provide the latter. She accepted the pupils' solutions even though they were wrong or invalid. However, Alice's pupils were active, and she gave them freedom to work on the problems themselves.

Bea emphasized the role of mathematics in the interviews. She strived to connect the tasks to the curriculum and wanted her pupils to succeed in these tasks. In her teaching, she checked the concepts and definitions in advance and decided on the kind of hands-on materials that would best serve the task. She ensured the pupils' understanding of the key concepts and checked that each pupil knew how to begin the task. She tried to deepen the pupils' thinking and get them to look for regularities. Although she prepared the lesson thoroughly and organized the work so that it would proceed smoothly, she guided the pupils' work throughout the lesson.

In the lesson concerning the "*Largest rectangle task*" the differences between Alice and Bea emerged quite clearly. The task was given as follows:

> *Draw various rectangles that have a perimeter of 30 cm. Calculate the areas of these rectangles. Which rectangle has the largest surface area? Can you find any systematics?*

Alice had not thought about the solution process before the lesson and had only partly checked the mathematical content. She did not remember how to calculate the area by using multiplication. Fortunately, during the lesson one of the pupils could explain it. Alice gave her pupils ordinary sheets with 7 mm squares so that the pupils had to use rulers to draw the rectangles. Because of this, some of the pupils began to use half centimetres and succeeded in finding the largest rectangle.

Bea had tried to check everything as usual but had not realized the possibility of a square. She had chosen drawing sheets with 1 cm × 1 cm squares. Her pupils were easily able to draw rectangles and calculate areas by counting the squares, but they got stuck with whole centimetres, and thus failed to find the largest area.

To summarize, while Alice emphasized autonomy, Bea placed mostly focus on the mathematical content. If we generalize, we can conclude that in Alice's class, the pupils could reach their best but in Bea's class, the pupils reached the level that their teacher could reach. In many other lessons, Bea's pupils reached better solutions but Alice's pupils invented solutions that were perhaps more creative. Therefore, the study results emphasize the importance of preparing the lesson in order to be able to guide the pupils, but also the importance of giving the pupils freedom to invent and use their own imagination.

### Summary

As seen in previous examples, teacher's role is important during different phases of a problem-solving lesson. In launch phase, it is central that the teacher introduces the problem, central concepts and aims for the lesson, and motivates the pupils to work with the problem. By doing this, pupils are able to reach higher-level solutions. During explore phase, the teacher needs to give pupils cognitive support by guiding pupils with good questions and emotional support by encouraging them when they are frustrated. All support is important. It makes pupils to work harder and reach better results. Activating support provides best results because it helps pupils to expand their thinking.

## THE CONNECTION BETWEEN TEACHER'S ACTIONS AND PUPILS' AFFECTIVE REACTIONS

In this section, I will present one study I have conducted with my colleagues to articulate the connection between teacher's actions and pupils' affective reactions. The article was published in the International Journal of Science and Mathematics Education (Laine, Ahtee, & Näveri, 2019), and considers how the pupils experience the emotional atmosphere during mathematics lesson. Therefore, it gives another perspective to the relationship between teacher's actions and pupils' learning. The article focuses on factors in teachers' and pupils' behavior that could explain differences in the emotional atmosphere between the different classrooms. Here, the pupils' drawings have been used as a method to approach this problem.

The emotional atmosphere in a classroom has two levels: individual and classroom level (Hannula, 2011). The individual level looks at the individual experiences that occur in the class, whereas the classroom level looks at the class in terms of social interaction, communication and norms. There are also two temporal aspects of "affect": state and trait (Hannula, 2011). State refers to the emotional atmosphere at a specific moment in the class while trait refers to more long-term conditions.

Teachers have a central role in advancing social interaction and a positive atmosphere in their classes. Especially the emotional relationship between the teacher and the pupils is very important (Evans, Harvey, Buckley, & Yan, 2009) as it advances both pupils' social accommodation and their orientation to school (Harrison, Clarke, & Ungerer, 2007). In addition, positive friendships are important. Several studies have found a close connection between the atmosphere in the classroom, and emotional and social experiences (e.g., Frenzel, Pekrun, & Goetz, 2007).

The data is based on third and fifth graders' drawings collected at the beginning of the 2010 autumn term and end of the 2013 spring term in Finland (Helsinki area). The task for the pupils was as follows: "*Draw your teaching group, your teacher and the pupils, in a mathematics lesson. Use speaking and thinking bubbles to describe discussion and thinking. And show yourself as 'me' in your drawing*."

The evaluation of classroom emotional atmosphere was based on all pupils' and teacher's visible moods as well as their speech and thought bubbles in the drawings. The classroom emotional atmosphere was classified into three categories: 1) *positive*, when all the drawn pupils and the teacher smile and/or think positively, some can be neutral; 2) *negative*, when all the drawn persons are sad or angry or think negatively, some can be neutral; 3) *other*, when all facial or other expressions are neutral, ambivalent or unidentifiable. Examples of analyses are presented in Laine et al. (2013) and Laine et al. (2015).

In the article we concentrated on Claire's and Fiona's classes where the emotional atmosphere changed to more positive according to the pupils' drawings, and, respectively, on Daisy's and Helen's classes where the atmosphere changed to more negative from third to fifth grade (see Table 8).

| | | Claire | Fiona | Daisy | Helen |
|---|---|---|---|---|---|
| 3rd grade | Positive | 47% (9) | 29% (5) | 44% (8) | 36% (4) |
| | Negative | 11% (2) | 0% | 0% | 9% (1) |
| | Other | 42% (8) | 71% (12) | 55% (10) | 55% (6) |
| 5th grade | Positive | 58% (11) | 42% (8) | 33% (6) | 35% (6) |
| | Negative | 0% | 5% (1) | 11% (2) | 24% (4) |
| | Other | 42% (8) | 53% (10) | 56% (10) | 41% (7) |

*Table 8.* The distribution of the emotional atmosphere in the third grade and fifth grade in Claire's, Fiona's, Daisy's and Helen's classes.

In the pupils' drawings from Claire's, Fiona's, Daisy's, and Helen's third and fifth grade classes, we looked for features that could explain the differences in the emotional atmosphere in the fifth grade. Table 9 shows the number of the pupils whose drawings contained following pupil actions: pupils are asking for help from the teacher or from their classmates, pupils are sitting alone at their desks, and pupils are talking to each other about mathematics. Table 9 also includes the number of pupils whose drawings contained following teacher actions: the teacher is located close to the pupils, the teacher is helping or encouraging the pupils, and the teacher is praising or criticizing the pupils.

| | Emotional atmosphere more positive in the fifth grade | | | | Emotional atmosphere more negative in the fifth grade | | | |
|---|---|---|---|---|---|---|---|---|
| Teacher | Claire | | Fiona | | Daisy | | Helen | |
| Grade | 3rd | 5th | 3rd | 5th | 3rd | 5th | 3rd | 5th |
| Number of pupils | 19 | 19 | 17 | 19 | 18 | 18 | 11 | 17 |
| Pupils are asking for help. | 15 | 10 | 7 | 9 | 1 | 1 | 1 | 3 |
| Pupils are sitting alone. | 3 | 4 | 1 | 1 | 8 | 15 | 5 | 8 |
| Pupils are talking about mathematics. | 9 | 10 | 4 | 6 | 0 | 0 | 0 | 1 |
| The teacher is close to the pupils. | 6 | 8 | 0 | 4 | 1 | 0 | 2 | 3 |
| The teacher is helping or encouraging the pupils. | 6 | 7 | 3 | 5 | 0 | 0 | 1 | 2 |
| The teacher is praising the pupils. | 2 | 1 | 2 | 1 | 5 | 3 | 0 | 3 |
| The teacher is criticizing or embarrassing the pupils. | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 5 |

*Table 9.* The distribution of the explaining factors.

In Claire's and Fiona's classrooms the pupils drew themselves asking for more help than in Daisy's and Helen's classrooms. Likewise, in Claire's and Fiona's classrooms the pupils drew themselves discussing mathematics with each other more often than in Daisy's and Helen's classrooms. On the other hand, in Daisy's and Fiona's classrooms the pupils drew themselves more frequently sitting alone at their desks than in Claire's and Fiona's classrooms. Interaction among the pupils seems to be very important. An open and tolerant atmosphere is seen from the drawings of classrooms with a positive emotional atmosphere (Claire and Fiona). In an open atmosphere the pupils are used to telling and arguing about their own views (Newstead, 1998), and showing their lack of knowledge by asking for help (Ryan, Gheen, & Midgley, 1998). The teacher has a central role in constructing the emotional atmosphere during mathematics lessons (Evans et al., 2009; Harrison et al., 2007). In Claire's and Fiona's classes the teacher was drawn to be near the pupils helping them and no student drew the teacher criticizing the pupils.

## CONCLUSIONS

I will now get back to my title: How to promote learning in problem-solving? Based on my understanding one fundamental element is safe and supportive emotional atmosphere. Interaction between the pupils seems to be of crucial importance in the development of the positive emotional atmosphere. The atmosphere in the classroom should be such that learning is appreciated and it allows the pupils to show their own incomprehension or lack of knowledge (Ryan, Gheen, & Midgley, 1998). The teacher has a central role in constructing the emotional atmosphere during mathematics lessons (Evans et al., 2009; Harrison et al., 2007). Especially, the emotional relationship between the teacher and the pupils, and the teacher's skill in evaluating and responding to pupils' feelings affect the emotional atmosphere (Evans et al., 2009). Teacher should be near to the pupils and help and encourage them to ponder and explain their thinking.

Pupils also need cognitive support, i.e., support to solve the problem and advance their thinking. Activating support based on pupils' solutions and good questions help pupils to deepen their thinking (Sahin & Kulm, 2008). Pupils need time to think and invent their own solutions. It is very harmful if the teacher reveals solutions because she/he spoils the joy of inventing. Teachers need to evaluate all the time what is enough help and balancing between their own visions and pupils creativity is challenging. Guiding problem-solving is a very complex task and the teachers need time to practice it.

## References


Evans, I. M., Harvey, S. T., Buckley, L., & Yan, E. (2009). Differentiating classroom climate concepts: Academic, management, and emotional environments. *New Zealand Journal of Social Sciences* *4*(2), 131–146. doi: http://dx.doi.org/10.1080/1177083X.2009.9522449

Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Perceived learning environment and students' emotional experiences: a multilevel analysis of mathematics classrooms. *Learning and Instruction*, *17*(5), 478–493.

Hannula, M. S. (2015). Emotions in problem solving. In S. J. Cho (Ed.), Selected regular lectures from the 12th International Congress on Mathematical Education ( pp. 269–288). Cham, Germany: Springer.

Harrison, L. J., Clarke, L., & Ungerer, J. A. (2007). Children's drawings provide a new perspective on teacher-child relationship quality and school adjustment. *Early Childhood Research Quarterly*, *22*(1), 55–71. doi: http://dx.doi.org/10.1016/j.ecresq.2006.10.003

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.

Laine, A. Ahtee, M. & Näveri, L. (2019). Impact of teacher's actions on emotional atmosphere in mathematics lessons in primary school. *International journal of mathematics and science education*. https://doi.org/10.1007/s10763-018-09948-x

Laine, A., Näveri, L., Ahtee, M., & Pehkonen, E. (2016). Opettajan toiminnan yhteys kolmasluokkalaisten onnistumiseen ongelmatehtävän ratkaisemisessa. *NMI-bulletin, 26*(2), 43–55.

Laine, A., Ahtee, M., Näveri, L., Pehkonen, E., & Hannula, M. (2018). Teachers' influence on the quality of pupils' written explanations – Third-graders solving a simplified arithmagon task during a mathematics lesson. *LUMAT: International Journal on Math, Science and Technology Education, 6*(1), 87–104. https://doi.org/10.31129/LUMAT.6.1.255

Laine, A., Näveri, L., Ahtee, M., Hannula, M. S., & Pehkonen, E. (2013). Emotional atmosphere in third-graders' mathematics classroom – an analysis of pupils' drawings. *Nordisk matematikdidaktik, 17*(3-4), 41–56.

Laine, A., Näveri, L., Ahtee, M., Pehkonen, E., & Hannula, M. S. (2017). Connections of primary teachers' actions and pupils' solutions to an open problem. *International Journal of Science and Mathematics Education, 16*(5), 967-983. doi: 10.1007/s10763-017-9809-3

Laine, A., Ahtee, M., Näveri, L. M., Pehkonen, E., Portaankorva-Koivisto, P., & Tuohilampi, L. (2015). Collective emotional atmosphere in mathematics lesson based on Finnish fifth graders' drawings. *LUMAT: Research and Practice in Math, Science and Technology Education, 3*(1), 87–100.

Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. London, England: Addison-Wesley.

National Board of Education. (2004). *National Core Curriculum for Basic Education 2004*. Finnish National Board of Education. Vammala, Finland: Vammalan Kirjapaino Oy.

Newstead, K. (1998). Aspects of children's mathematics anxiety. *Educational studies in mathematics, 36*(1), 53–71.

Pehkonen, E. (1997). Use of problem fields as a method for educational change. In E. Pehkonen (Ed.), *Use of open-ended problems in mathematics classroom* (pp. 73–84). University of Helsinki. Department of Teacher Education. Research Report 176.

Pekrun, R., & Stephens, E. J. (2010). Achievement emotions: A control-value approach. *Social and Personality Psychology Compass*, 4, 238–255.

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Portaankorva-Koivisto, P., Laine, A. & Ahtee, M. (under review). Two primary teachers – Two different mathematical problem-solving instructors. *International Journal for Mathematics Teaching and Learning*.

Ryan, A. M., Gheen, M. H., & Midgley, C. (1998). Why do some students avoid asking for help? An examination of the interplay among students' academic efficacy, teachers' social–emotional role, and the classroom goal structure. *Journal of Educational Psychology, 90*(3), 528–535. doi: http://dx.doi.org/10.1037/0022-0663.90.3.528

Sahin, A., & Kulm, G. (2006). Sixth grade mathematics teachers' intentions and use of probing, guiding, and factual questions. *Journal of Mathematics Teacher Education, 11*(3), 221–241. Schoenfeld A.H. (1985). *Mathematical problemsolving*. Orlando, FL: Academic Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem-solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York, NY: MacMillan.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning, 10*, 313–340.

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership,* 61(5), 12–17.

Yeo, J. B. (2017). Development of a framework to characterise the openness of mathematical tasks. *International Journal of Science and Mathematics Education*, 15(1), 175–191.